COEXISTENCE OF ORDERED AND CHAOTIC STATES IN NUCLEAR STRUCTURE

V.R. Manfredi[1,2], L. Salasnich[2,3]

[1] Dep. of Physics, Univ. of Padova, Italy, [2] INFN, Padova, Italy,
[3] Dep. of Pure and Appl. Math., Univ. of Padova, Italy

1   Introduction

In atomic nuclei, as in other many–body systems, the classical phase space is mixed, so ordered and chaotic states generally coexist [1]. In this contribution we discuss some models, showing the transition from order to chaos. In several cases a clear correspondence between classical and quantum chaos has been established. In particular the transition from ordered to chaotic states will be discussed in the framework of the shell and roto–vibrational models. The spectral statistics of low–lying states of several fp shell nuclei are studied with realistic shell–model calculations [2]. Furthermore, for the roto–vibrational model the Gaussian curvature criterion of the potential energy clearly shows the transition from order to chaos for different values of rotational frequency [3].

2   Shell Model Calculations

In this section we discuss the statistical analysis of the shell–model energy levels in the $A = 46$–50 region. By using second–quantization notation, the nuclear shell–model Hamiltonian can be written as

$$H = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^{+} a_{\alpha} + \sum_{\alpha\beta\gamma\delta} < \alpha\beta|V|\delta\gamma > a_{\alpha}^{+} a_{\beta}^{+} a_{\gamma} a_{\delta} , \qquad (1)$$

where the labels denote the accessible single–particle states, ϵ_{α} is the corresponding single–particle energy, and $< \alpha\beta|V|\delta\gamma >$ is the two–body matrix element of the nuclear residual interaction.

Exact calculations are performed in the $(f_{7/2}, p_{3/2}, f_{5/2}, p_{1/2})$ shell–model space, with ^{40}Ca as an inert core, by using a fast implementation of the Lanczos algorithm with the code ANTOINE. The interaction we use is a minimally modified Kuo–Brown realistic force with monopole improvements [2].

The spectral statistic $P(s)$ may be used to study the local fluctuations of the energy levels. $P(s)$ is the distribution of nearest–neighbour spacings $s_i = \tilde{E}_{i+1} - \tilde{E}_i$ of the

unfolded levels. To quantify the chaoticity of $P(s)$ in terms of a parameter, it can be compared to the Brody distribution,

$$P(s, \omega) = \alpha(\omega + 1)s^\omega \exp\left(-\alpha s^{\omega+1}\right) , \qquad (2)$$

with $\alpha = (\Gamma[\frac{\omega+2}{\omega+1}])^{\omega+1}$. This distribution interpolates between the Poisson distribution ($\omega = 0$) of regular systems and the Wigner distribution ($\omega = 1$) of chaotic ones. The parameter ω can be used as a simple quantitative measure of the degree of chaoticity.

The following table shows the calculated Brody parameter ω for the nearest neighbour level spacings distribution for $0 \leq J \leq 9$, $T = T_z$ states up to 4, 5 and 6 MeV above the yrast line in the analyzed nuclei.

Energy	^{46}V	^{46}Ti	^{46}Sc	^{46}Ca	^{48}Ca	^{50}Ca	^{46}V+^{46}Ti+^{46}Sc	^{46}Ca+^{48}Ca+^{50}Ca
≤ 4 MeV	1.14	0.90	0.81	0.41	0.58	0.67	0.92	0.56
≤ 5 MeV	1.10	0.81	0.96	0.53	0.58	0.69	0.93	0.60
≤ 6 MeV	0.93	0.94	0.99	0.51	0.66	0.62	0.95	0.61

We observe that Ca isotopes are less chaotic than their neighbours. In fact, the two–body matrix elements of the proton–neutron interaction are, on average, larger than those of the proton–proton and neutron–neutron interactions. Consequently, the single–particle mean–field motion in nuclei with only neutrons in the valence orbits suffers less disturbance and is thus more regular.

3   The Roto–Vibrational Model

The Hamiltonian of an axially symmetric roto–vibrational nucleus is given by [3]

$$H = \frac{1}{2}B(3a_0^2 + 2a_2^2)\omega^2 + \frac{1}{2}B(\dot{a}_0^2 + 2\dot{a}_2^2) + V(a_0, a_2) , \qquad (3)$$

with

$$V(a_0, a_2) = \frac{1}{2}C_2(a_0^2 + 2a_2^2) + \sqrt{\frac{2}{35}}C_3 a_0(6a_2^2 - a_0^2) + \frac{1}{5}C_4(a_0^2 + 2a_2^2)^2 + V_0 , \qquad (4)$$

where ω is the rotational frequency of the nucleus and V_0 is chosen to have the minimum of the potential equal to zero. The dynamical variables a_0 and a_2 are connected to the deformation β and asymmetry γ by the relations $a_0 = \beta \cos \gamma$ and $a_2 = \frac{\beta}{\sqrt{2}} \sin \gamma$.

As is well known, the order–chaos transition in systems with two degrees of freedom may be studied by means of the Gaussian curvature criterion of the potential energy. It is, however, important to point out that *in general* the curvature criterion guarantees only a *local instability* and should therefore be combined with the Poincarè sections [3]. The effective potential of the system is $W(a_0, a_2) = \frac{1}{2}B(3a_0^2 + 2a_2^2)\omega^2 + V(a_0, a_2)$. Owing to the symmetry properties of the effective potential W, our study may be restricted to the case $W(a_0, a_2 = 0)$. To apply the above criterion to our system, the sign of the Gaussian curvature K can be obtained by solving the equation:

$$K(a_0) = \frac{\partial^2 W}{\partial a_0^2}(a_0, a_2 = 0) = \frac{12}{5}C_4 a_0^2 - 6\sqrt{\frac{2}{35}}C_3 a_0 + (C_2 + 3B\omega^2) = 0 , \qquad (5)$$

whose discriminant Δ is given by:

$$\Delta = \frac{72}{35}C_2 C_4 (\chi - \frac{14}{3} - 14\frac{B\omega^2}{C_2}) , \tag{6}$$

where $\chi = C_3^2/(C_2 C_4)$. If $\Delta \leq 0$, the curvature K is always positive and the motion is regular. If $\Delta > 0$, there is a region of negative curvature and the motion may be chaotic.

For $C_2 > 0$ and $0 < \chi < 14/3$ (spherical nuclei), the curvature is positive and therefore the motion is regular for all ω. For $\chi \geq 14/3$ (spherical and deformed nuclei) and $0 \leq \omega < \sqrt{\frac{C_2}{14B}(\chi - \frac{14}{3})}$, the curvature is negative and chaotic motion may appear.

For $C_2 < 0$ (γ–unstable nuclei), there is a region with negative curvature for $0 \leq \omega < \omega_c$, where $\omega_c = \sqrt{\frac{C_2}{14B}(\chi - \frac{14}{3})}$ is the critical frequency of the system. It is noteworthy that the shape of the effective potential $W(a_0, 0)$ changes drastically as a function of ω. If ω increases, there is a transition from chaos to order, i.e. the region of chaotic motion decreases and becomes zero for $\omega > \omega_c$.

4   Conclusions

The shell model calculations show that for Ca isotopes there are significant deviations from the Wigner distribution of chaotic systems. Concerning the roto–vibrational model of atomic nuclei, a chaos–order transition occurs as a function of the angular frequency of the nucleus.

References

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